

# NONCE-BASED CRYPTOGRAPHY RETAINING SECURITY WHEN RANDOMNESS FAILS

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# WEAK RANDOMNESS

bugs and bad implementations

targeted attack(s)



**debian OpenSSL**  
insufficient entropy



**PlayStation 3**  
ECDSA randomness

**RSA Certificate Keys**  
coinciding prime factors [1]



**Netscape**  
insufficient entropy

**/dev/random**  
... is not robust [2]

**DUAL EC**



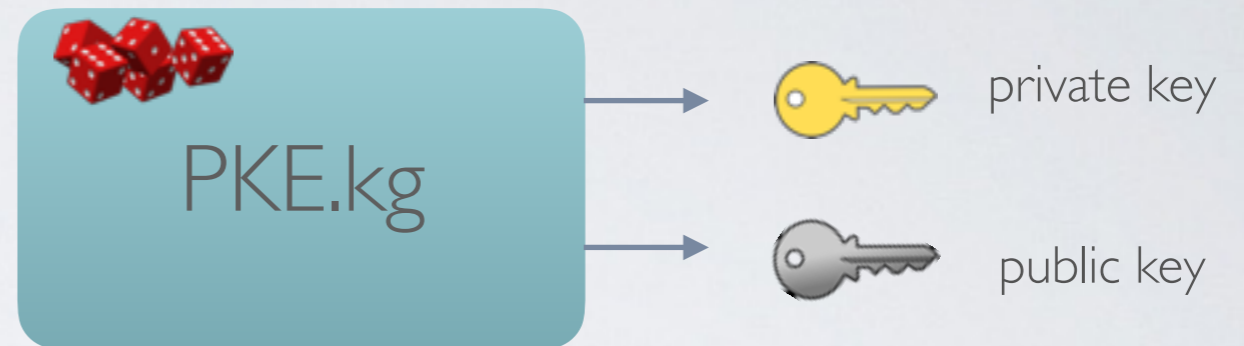
... and more?

[1; Heninger, Durumeric, Wustrow, Halderman, 2012; Lenstra, Hughes, Augier, Bos, Kleinjung, and Wachter, 2012]

[2; Dodis, Pointcheval, Ruhault, Vergnaud, Wichs, 2013]

# PUBLIC-KEY ENCRYPTION

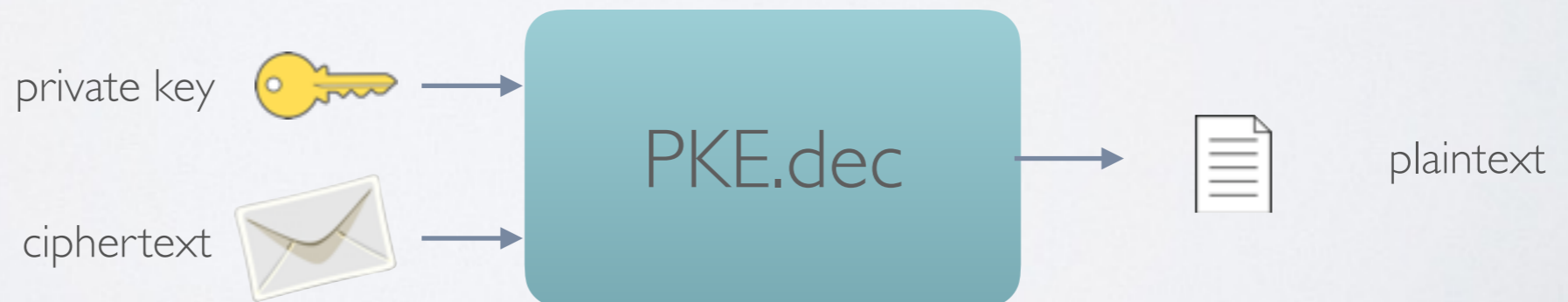
## 1. key generation



## 2. encryption

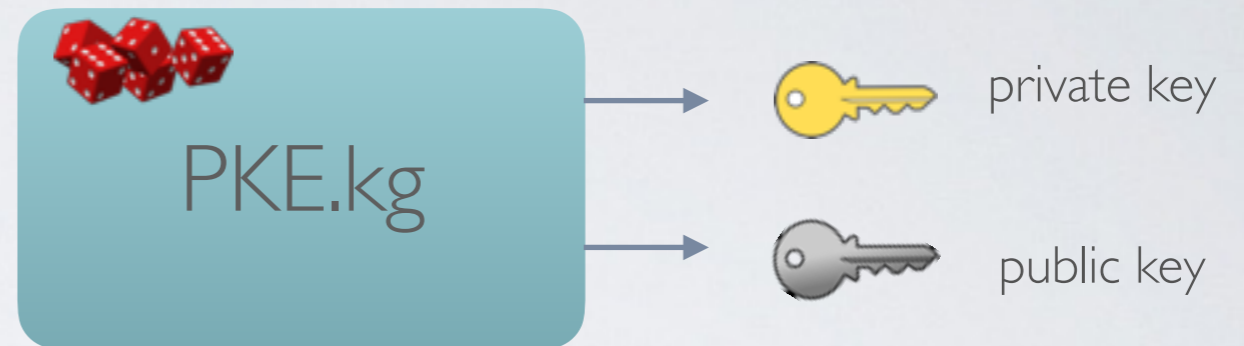


## 3. decryption

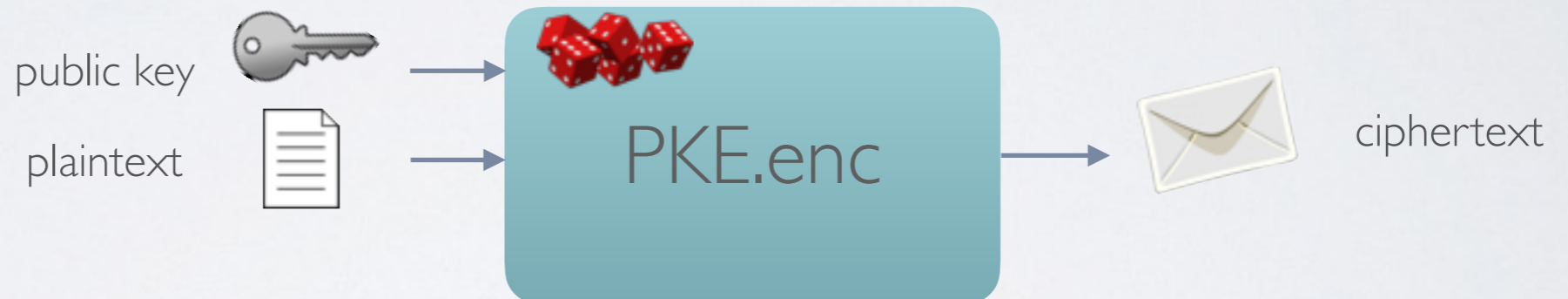


# PUBLIC-KEY ENCRYPTION

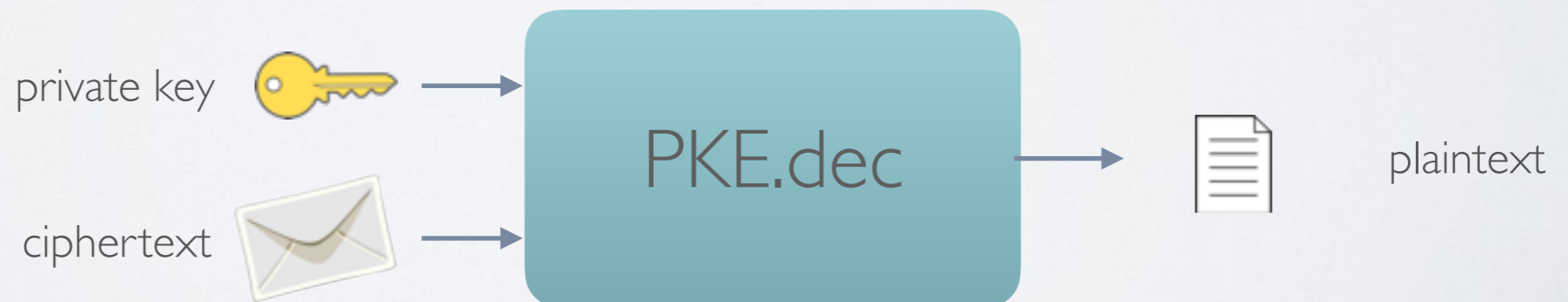
## 1. key generation



## 2. encryption



## 3. decryption



# PUBLIC-KEY ENCRYPTION

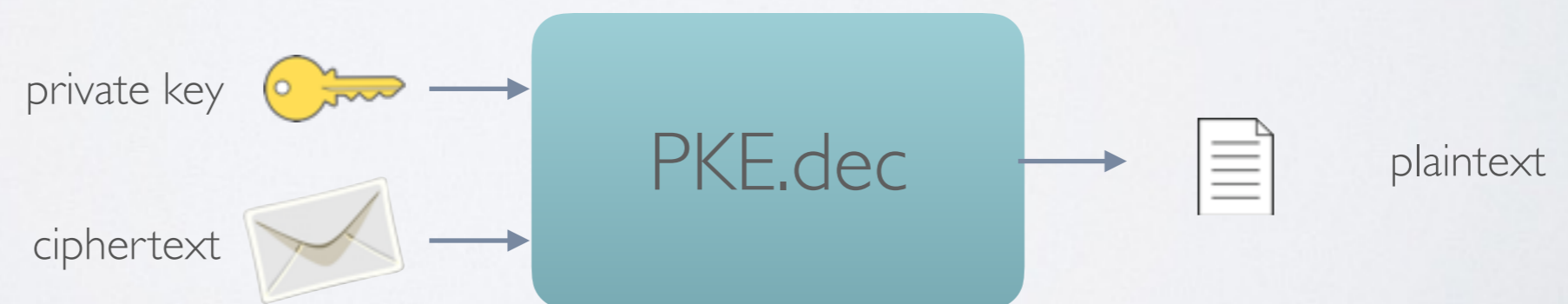
## 1. key generation



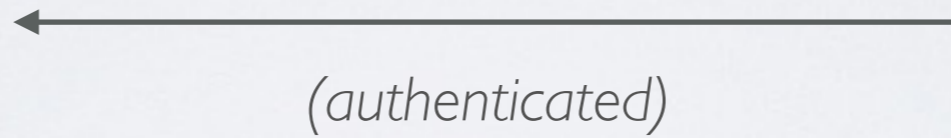
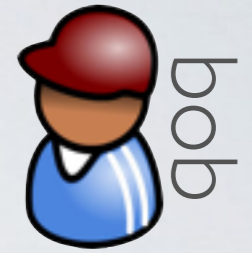
## 2. encryption



## 3. decryption



# USING PUBLIC-KEY ENCRYPTION

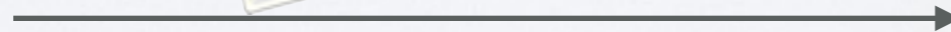


$$(sk, pk) \leftarrow \$ \text{PKE.kg}$$



$$m \leftarrow \text{PKE.dec}(sk, c)$$

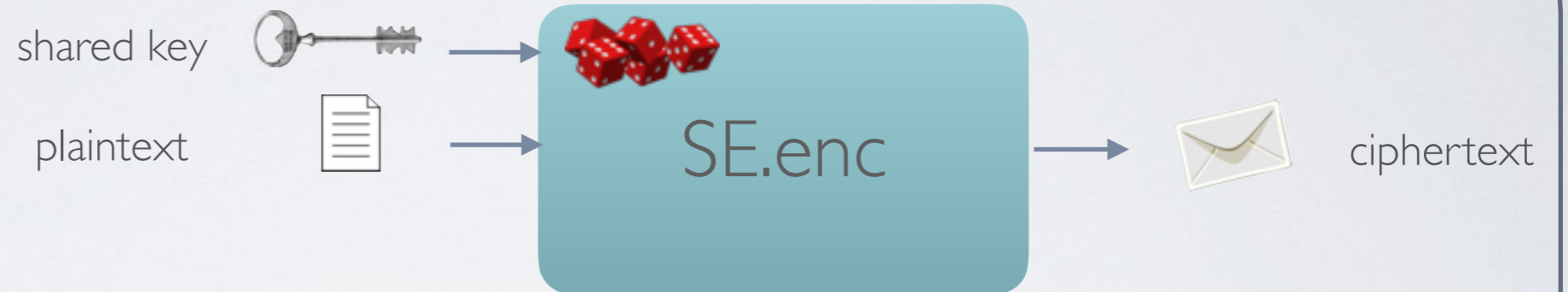
$$c \leftarrow \$ \text{PKE.enc}(pk, m)$$



$m$  message  
 $c$  ciphertext  
 $pk$  public key  
 $sk$  secret key

# SYMMETRIC ENCRYPTION AND NONCES

## 1. encryption



## 2. decryption



# SYMMETRIC ENCRYPTION AND NONCES

## 1. encryption



## 2. decryption





# SYMMETRIC ENCRYPTION AND NONCES

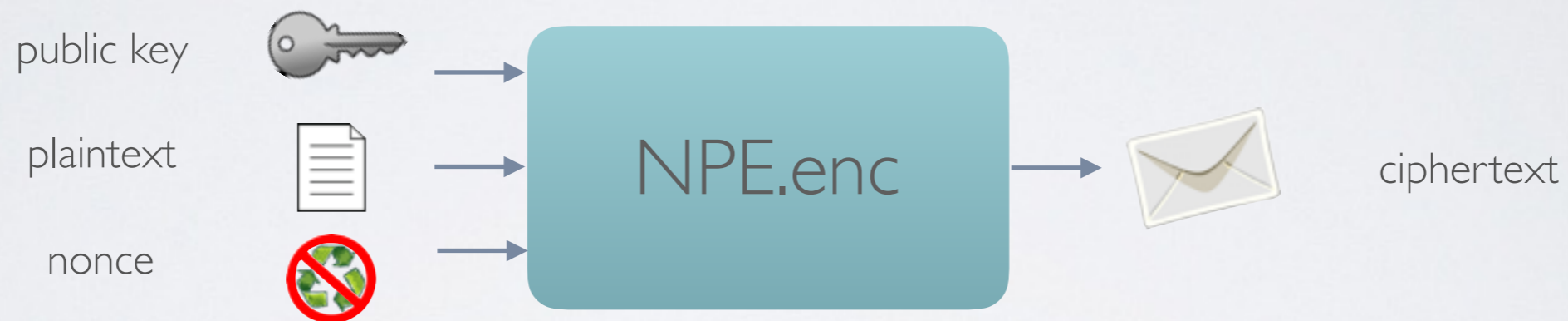
## 1. encryption



## 2. decryption



# WHAT ABOUT NONCE-BASED PKE?



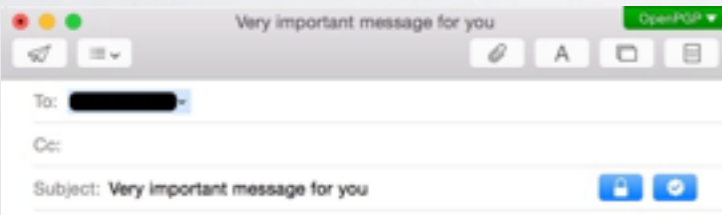
all input values may be known to an attacker!

# THE INTUITION



1. setup: generation of good random seed

2. keep state: sender stores seed  
**but** we hedge scheme against exposure



3. encryption: use seed along with nonce

# NONCE-BASED PKE

## 1a. receiver key generation

as before

## 1b. sender key generation

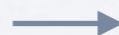
randomness



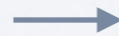
seed

## 2. encryption

public key



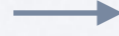
seed



plaintext



nonce



ciphertext

## 3. decryption

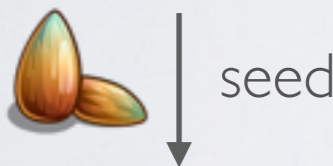
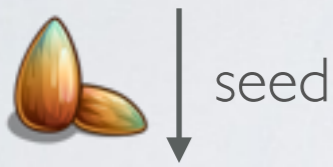
as before

# USING NONCE-BASED PKE



alice

seed  $\leftarrow$  NPE.skg

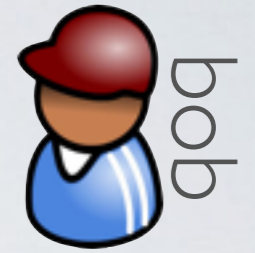


$c \leftarrow \text{NPE.enc}(pk, \text{seed}, m, \text{nonce})$

$m$  message  
 $c$  ciphertext  
 $pk$  public key  
 $sk$  secret key



(authenticated)



bob

$(sk, pk) \leftarrow$  NPE.rkg



$m \leftarrow \text{NPE.dec}(sk, c)$

# USING NONCE-BASED PKE



the sender has to keep state, but ...

1. same seed valid for multiple receivers
2. different seeds on, e.g., different devices
3. seeds can be updated at any time ... and
4. ... we are hedging against exposure of the seed

seed ←



$$c \leftarrow \text{NPE.enc}(pk, \text{seed}, m, \text{nonce})$$

NPE.rkg

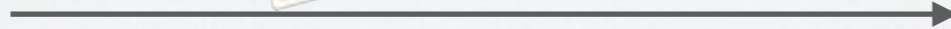


sk

$$m \leftarrow \text{NPE.dec}(sk, c)$$



c



*m* message  
*c* ciphertext  
*pk* public key  
*sk* secret key

# SECURITY GUARANTEES

security is guaranteed if **either**

sender seed secret

**and**

(nonce, message) pairs  
unique

**or**

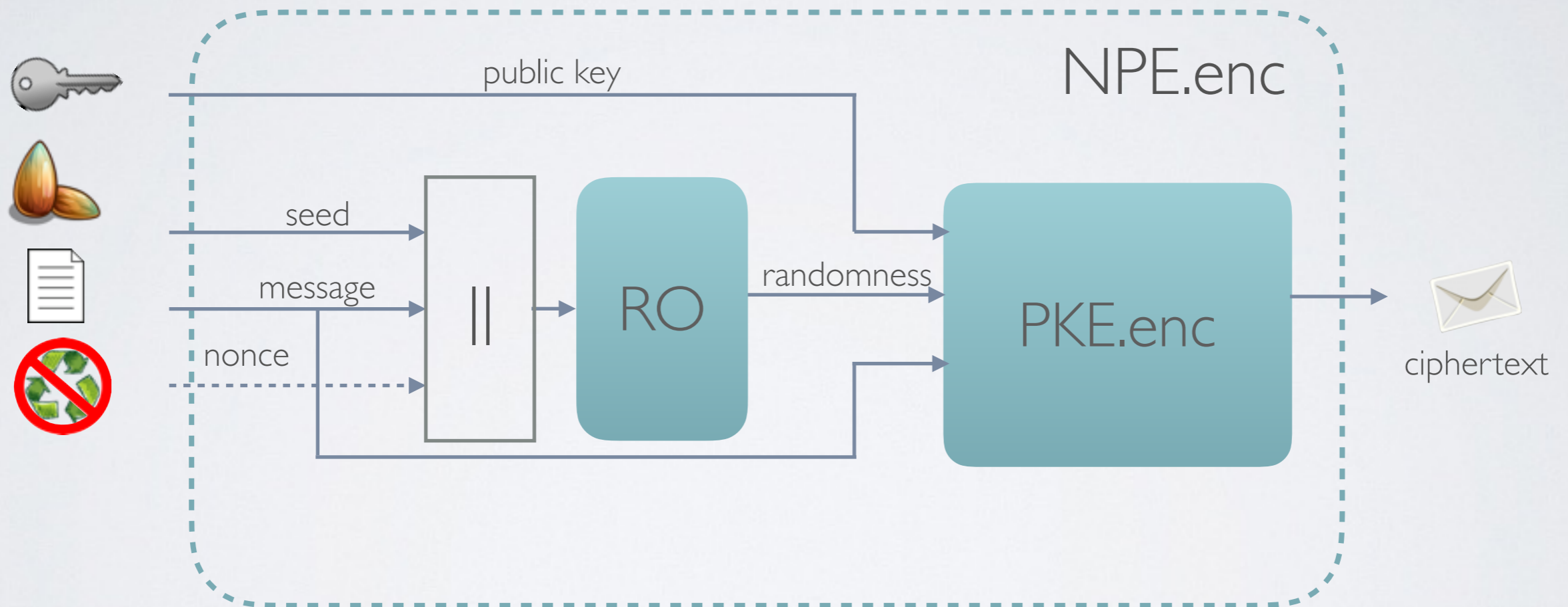
sender seed public

**and**

nonces secret  
and unpredictable.

include in nonces, e.g., sender and receiver addresses, time, system RNG output

# A RANDOM-ORACLE-BASED SCHEME



decryption remains unchanged



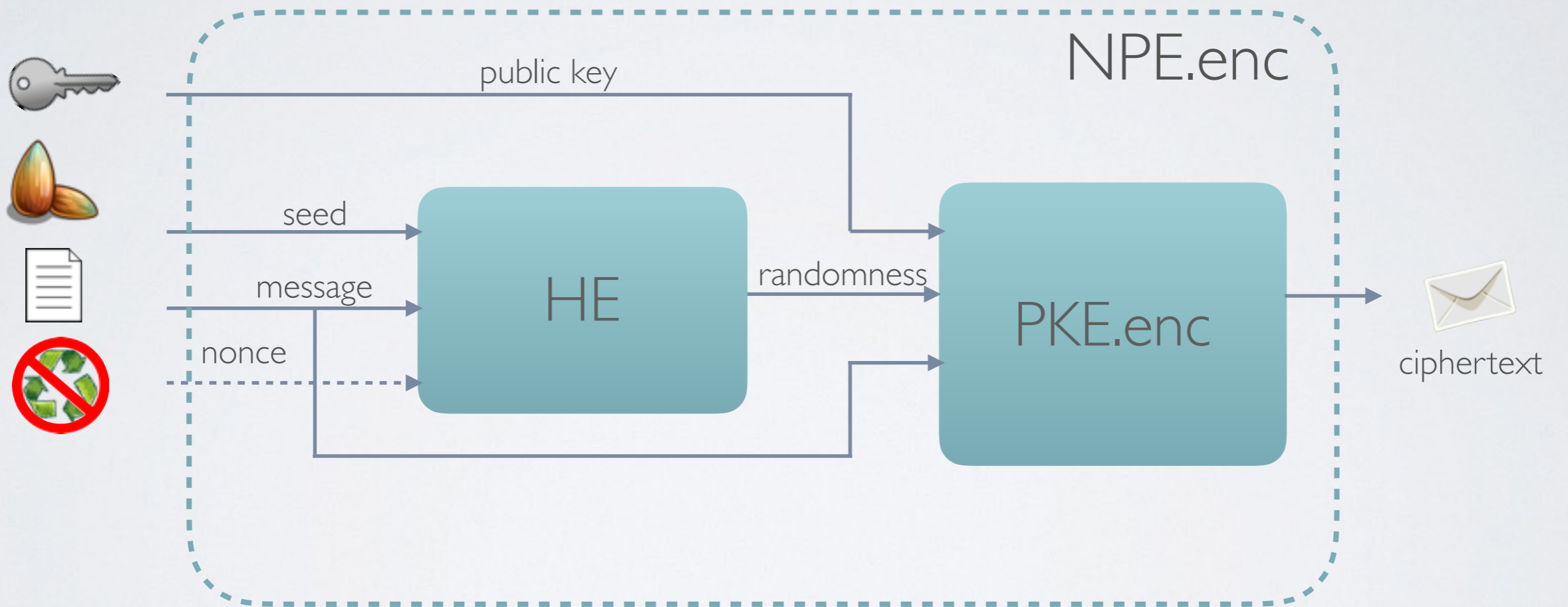
# MAIN TOOL: HEDGED EXTRACTORS



(a) PRF **if** seed is secret

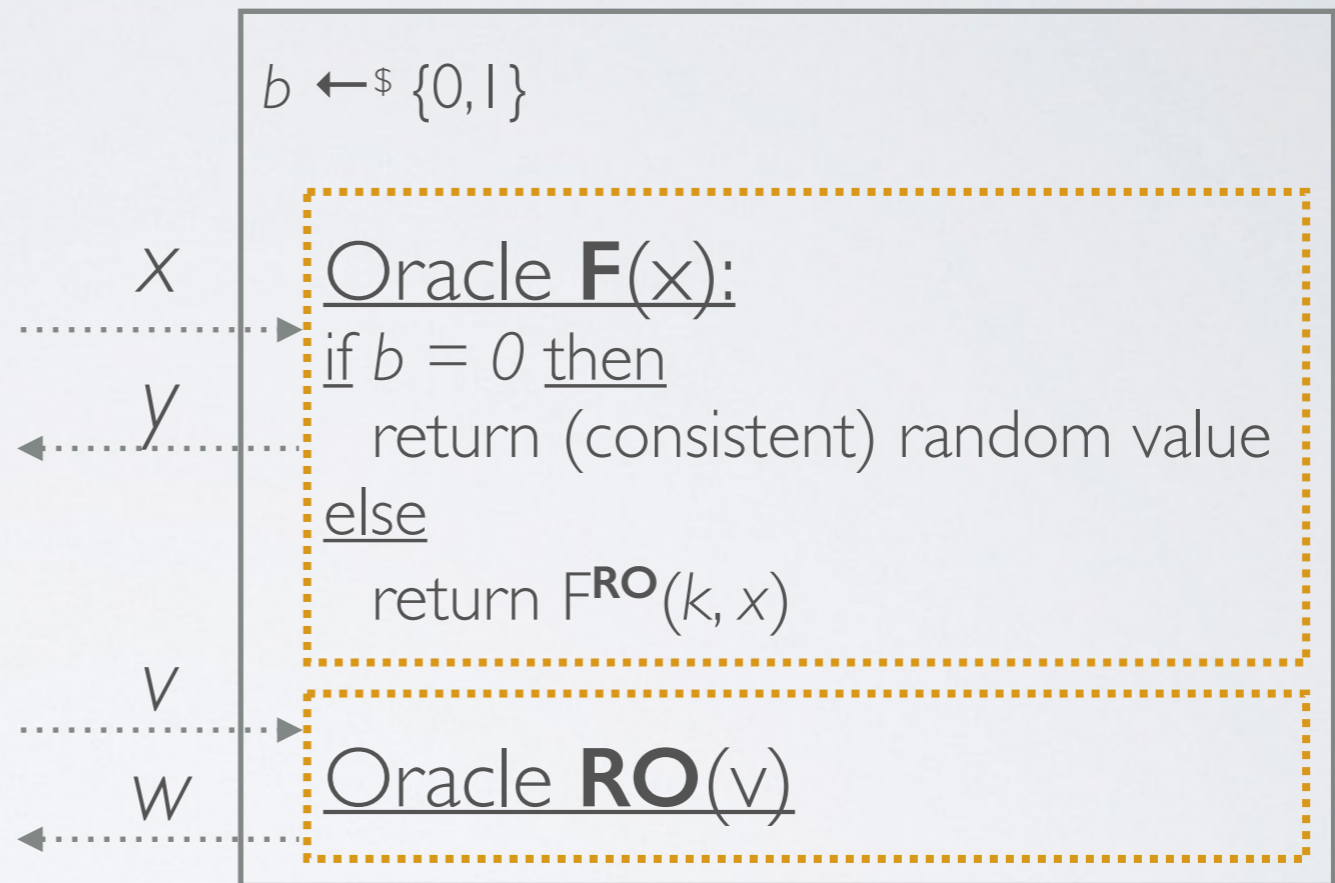
(b) strong extractor **if** seed public but random

# ADAPTING TO HEDGED-EXTRACTORS



# SECURITY I: PSEUDO-RANDOMNESS

$\mathcal{A}$

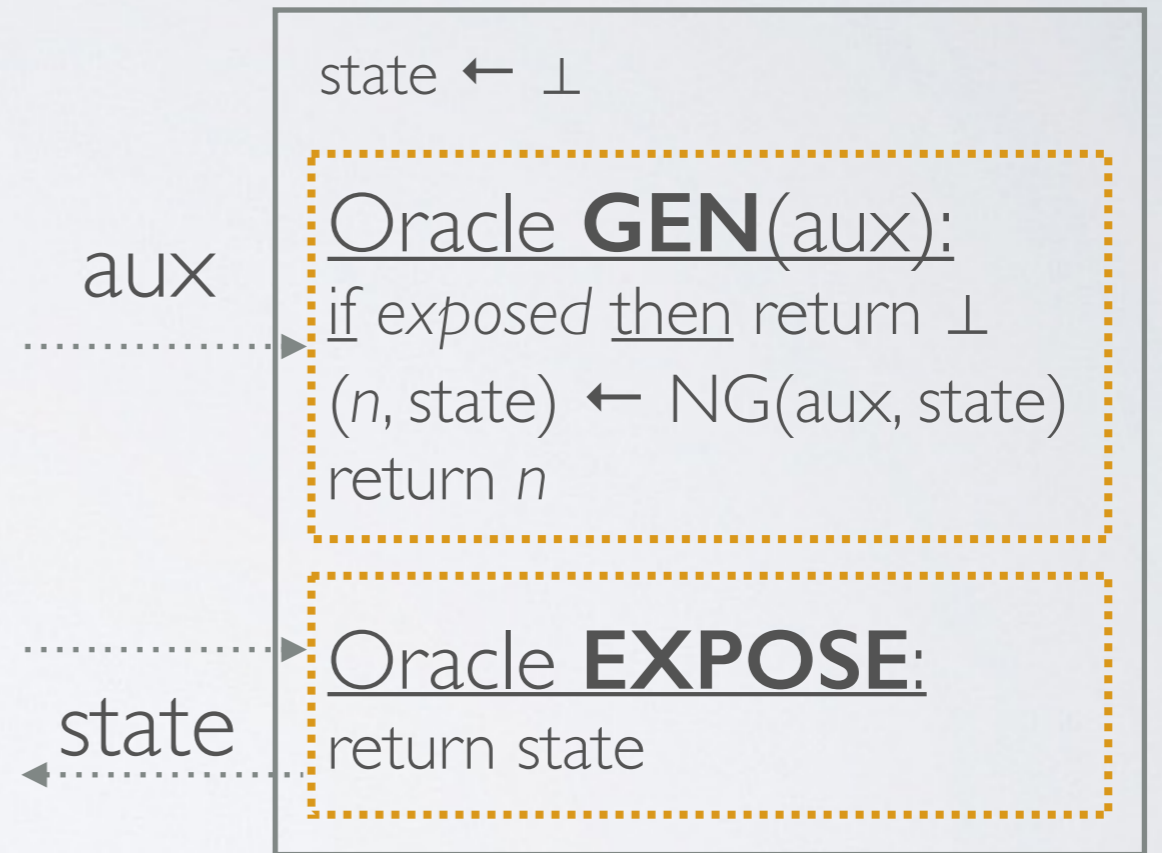


$$\text{Adv}^{\text{prf}}(\mathbf{F}, \mathcal{A}) = 2 \Pr [ b' \leftarrow_{\$} \mathcal{A}^{\mathbf{F}, \mathbf{RO}}; b = b' ] - 1$$

# (UNPREDICTABLE) NONCE GENERATORS



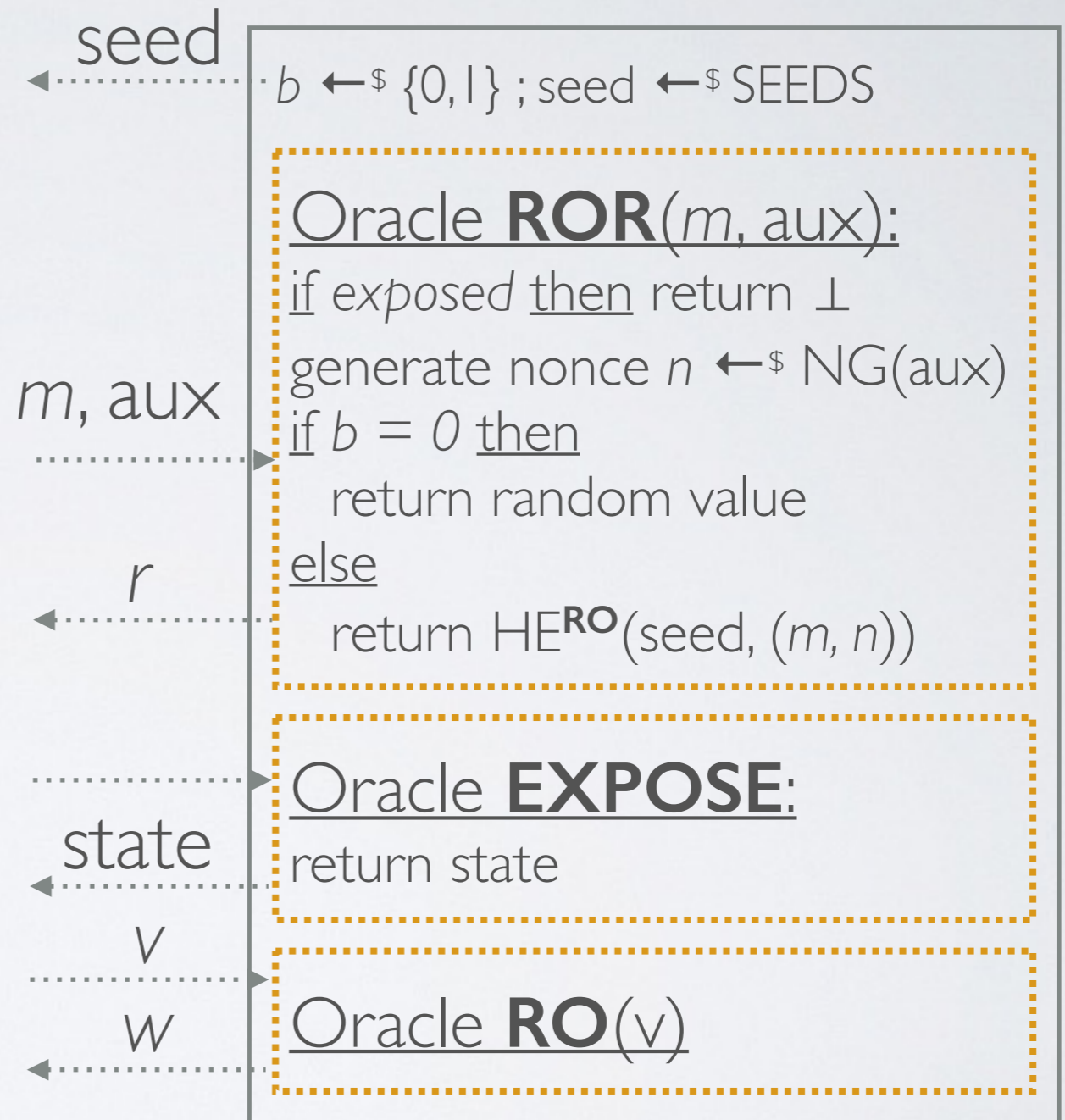
$\mathcal{A}$



$$\text{Adv}^{\text{pred}}(\text{NG}, \mathcal{A}) = \Pr [ n \leftarrow_{\$} \mathcal{A}^{\text{GEN, EXPOSE}}; n \in N \text{ or collision } ]$$

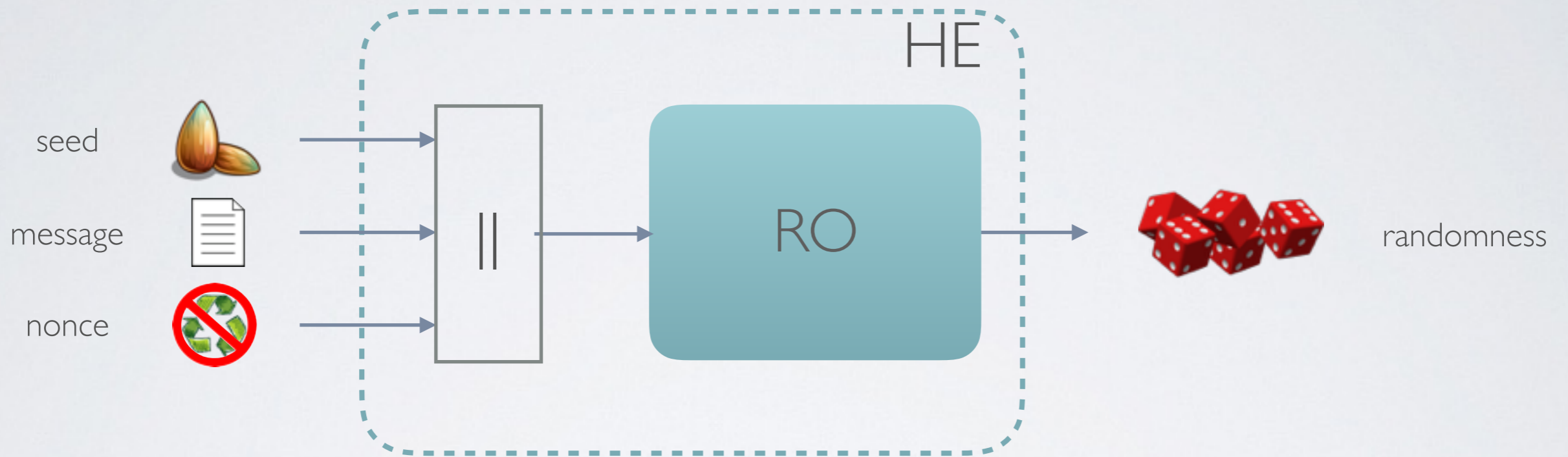
# SECURITY 2: EXTRACTION

$\mathcal{A}$



$$\text{Adv}^{\text{ror}}(\text{HE}, \text{NG}, \mathcal{A}) = 2 \Pr [ b' \leftarrow_{\$} \mathcal{A}^{\text{ROR, EXPOSE, RO}}; b = b' ] - 1$$

# THE RANDOM-ORACLE SCHEME



$$\text{Adv}^{\text{prf}}(\text{HE}, \mathcal{A}) \leq q \cdot 2^{-k}$$

$$\text{Adv}^{\text{ror}}(\text{HE}, \text{NG}, \mathcal{A}) \leq q \cdot \text{Adv}^{\text{pred}}(\text{NG}, \mathcal{B})$$

$q$  RO queries  
seed length  $k$

## RECALL: ALMOST-UNIVERSAL HASHING



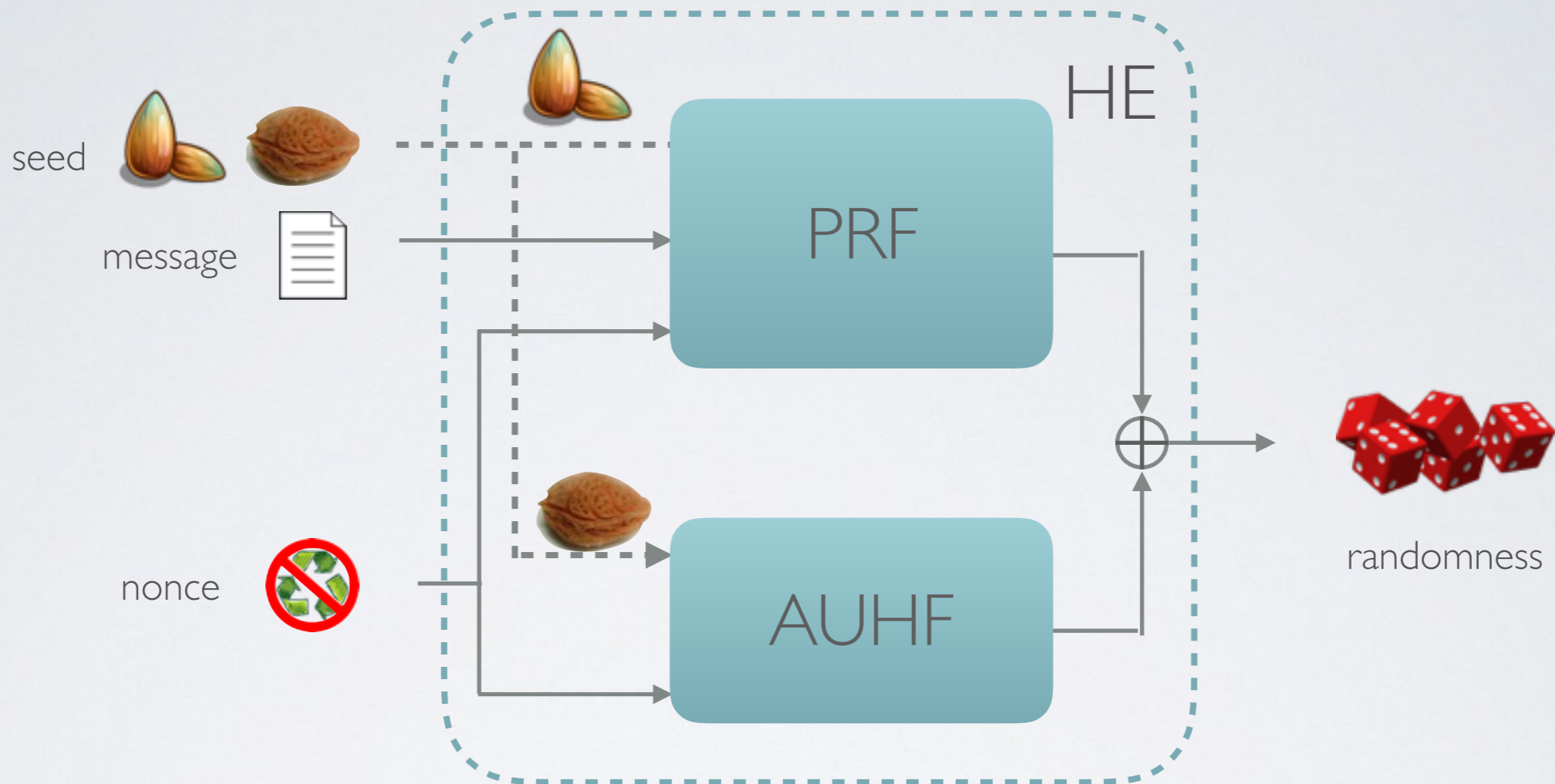
Definition:  $F: K \times X \rightarrow Z$  is  $\epsilon$ -AUHF if

$$\forall x \neq y: \Pr_k[ F(k, x) = F(k, y) ] \leq \epsilon$$

Leftover Hash Lemma: Let  $F$  be  $\epsilon$ -AUHF, then

$$k, z \approx_{\epsilon'(k)} k, F(k, x) \quad \text{with} \quad k \leftarrow_{\$} K; z \leftarrow_{\$} Z; x \text{ with min-entropy } k$$

# THE STANDARD-MODEL SCHEME



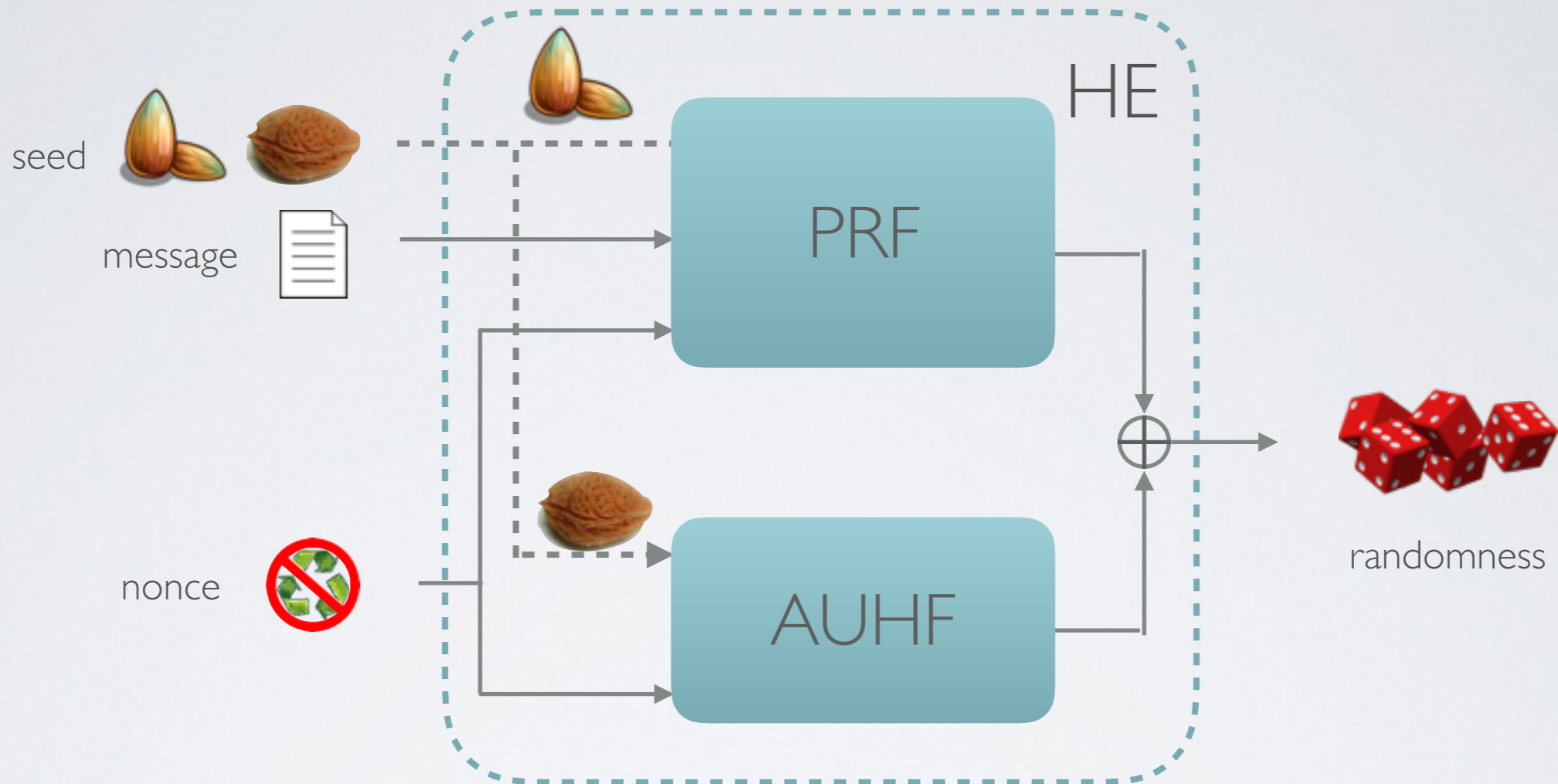
$$\text{Adv}^{\text{prf}}(\text{HE}, \mathcal{A}) \leq \text{Adv}^{\text{prf}}(\text{PRF}, \mathcal{B})$$

$$\text{Adv}^{\text{ror}}(\text{HE}, \text{NG}, \mathcal{A}) \leq q \cdot \varepsilon'(k)$$

$$\text{if } \text{Adv}^{\text{pred}}(\text{NG}, \mathcal{C}) \leq 2^{-k}$$



# THE STANDARD-MODEL SCHEME



caveat: nonces must be independent of seed

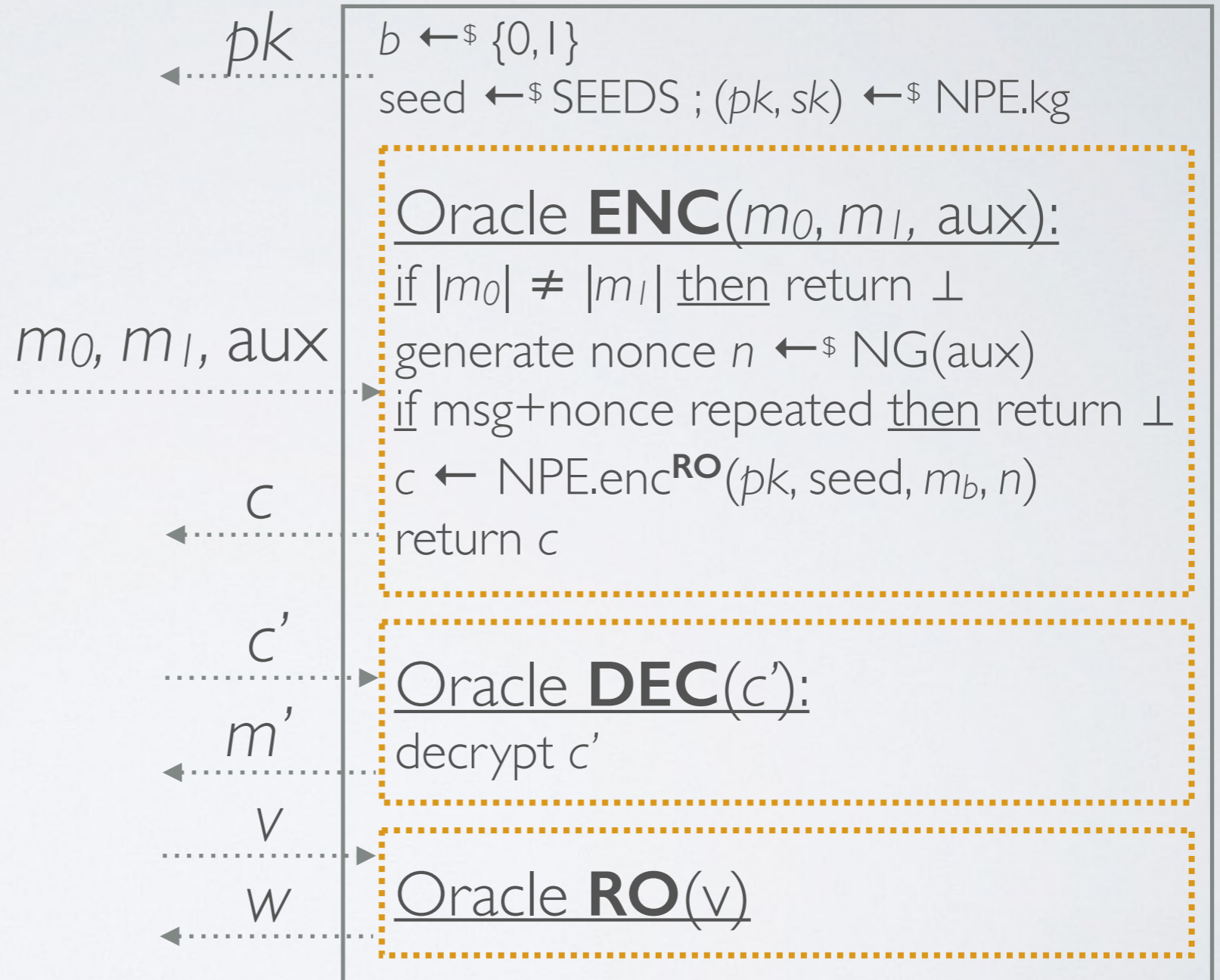
$$\text{Adv}^{\text{prf}}(\text{HE}, \mathcal{A}) \leq \text{Adv}^{\text{prf}}(\text{PRF}, \mathcal{B})$$

$$\text{Adv}^{\text{ror}}(\text{HE}, \text{NG}, \mathcal{A}) \leq q \cdot \varepsilon'(k)$$

$$\text{if } \text{Adv}^{\text{pred}}(\text{NG}, \mathcal{C}) \leq 2^{-k}$$

# NONCE-BASED PRIVACY, ONE

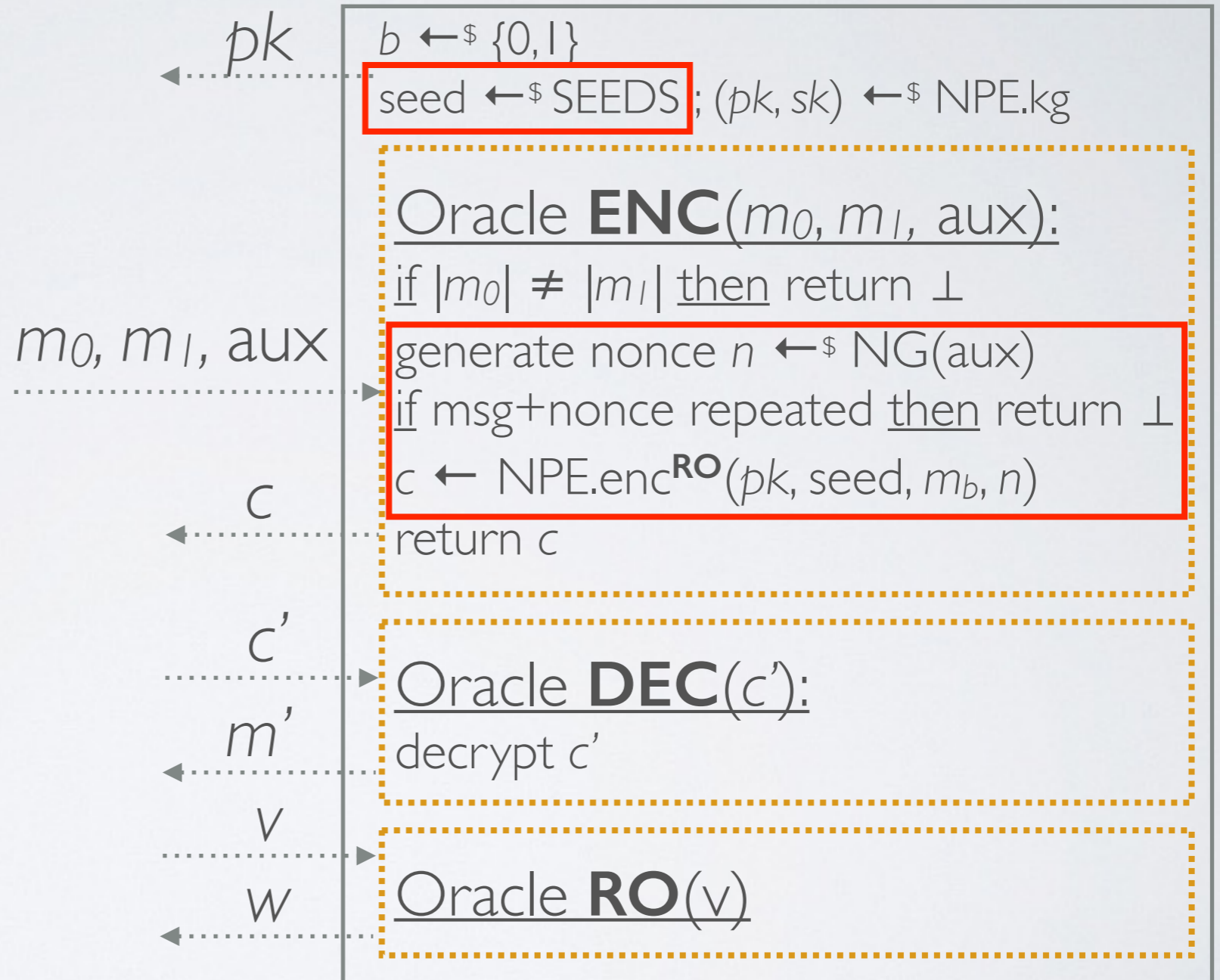
$\mathcal{A}$



$$\text{Adv}^{\text{nbpI}}(\text{NPE}, \mathcal{A}) = 2 \Pr [ b' \leftarrow_{\$} \mathcal{A}^{\text{ENC, DEC, RO}}; b = b' ] - 1$$

# NONCE-BASED PRIVACY, ONE

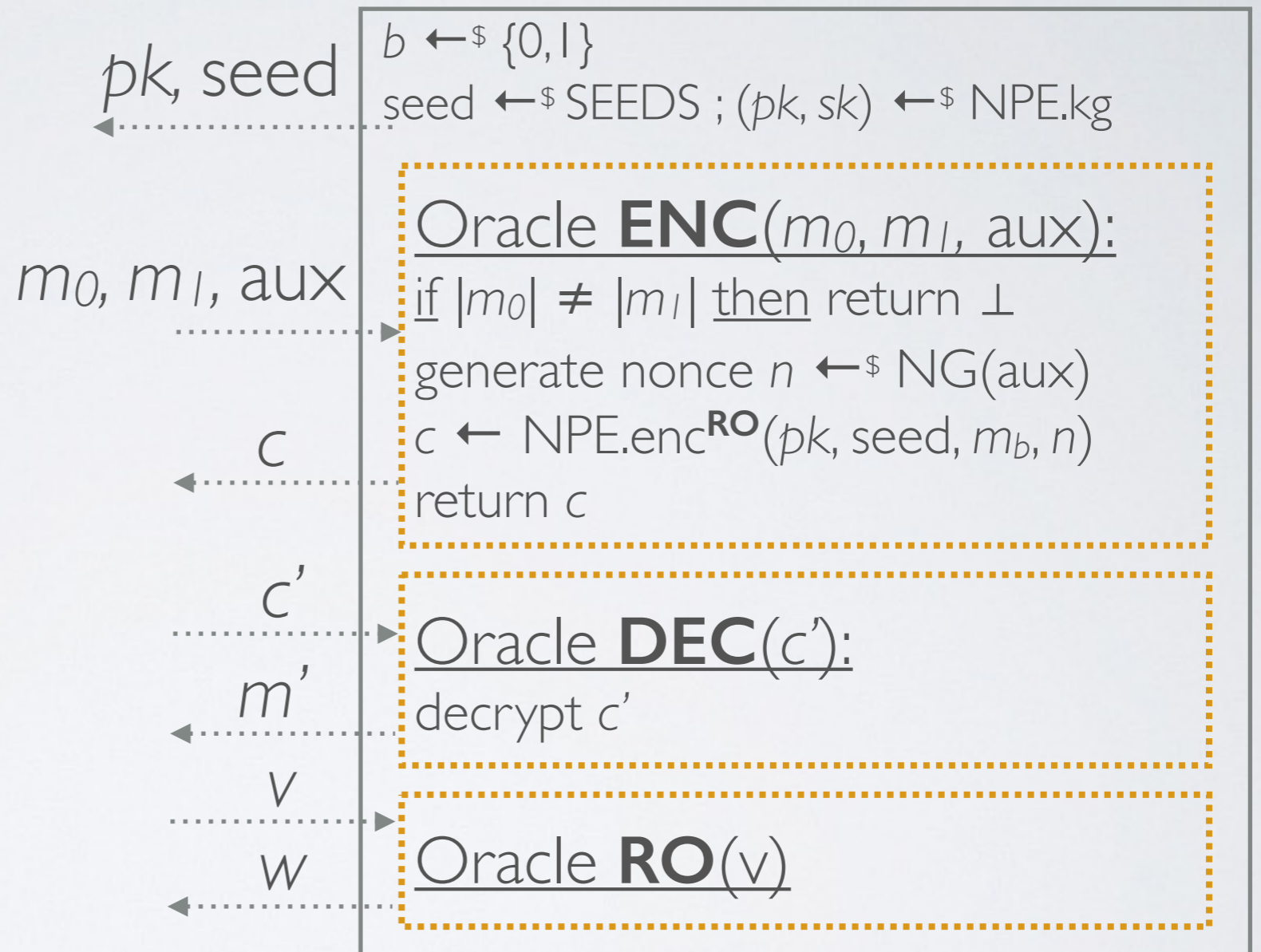
$\mathcal{A}$



$$\text{Adv}^{\text{nbpI}}(\text{NPE}, \mathcal{A}) = 2 \Pr [ b' \leftarrow_{\$} \mathcal{A}^{\text{ENC,DEC,RO}}; b = b' ] - 1$$

# NONCE-BASED PRIVACY, TWO

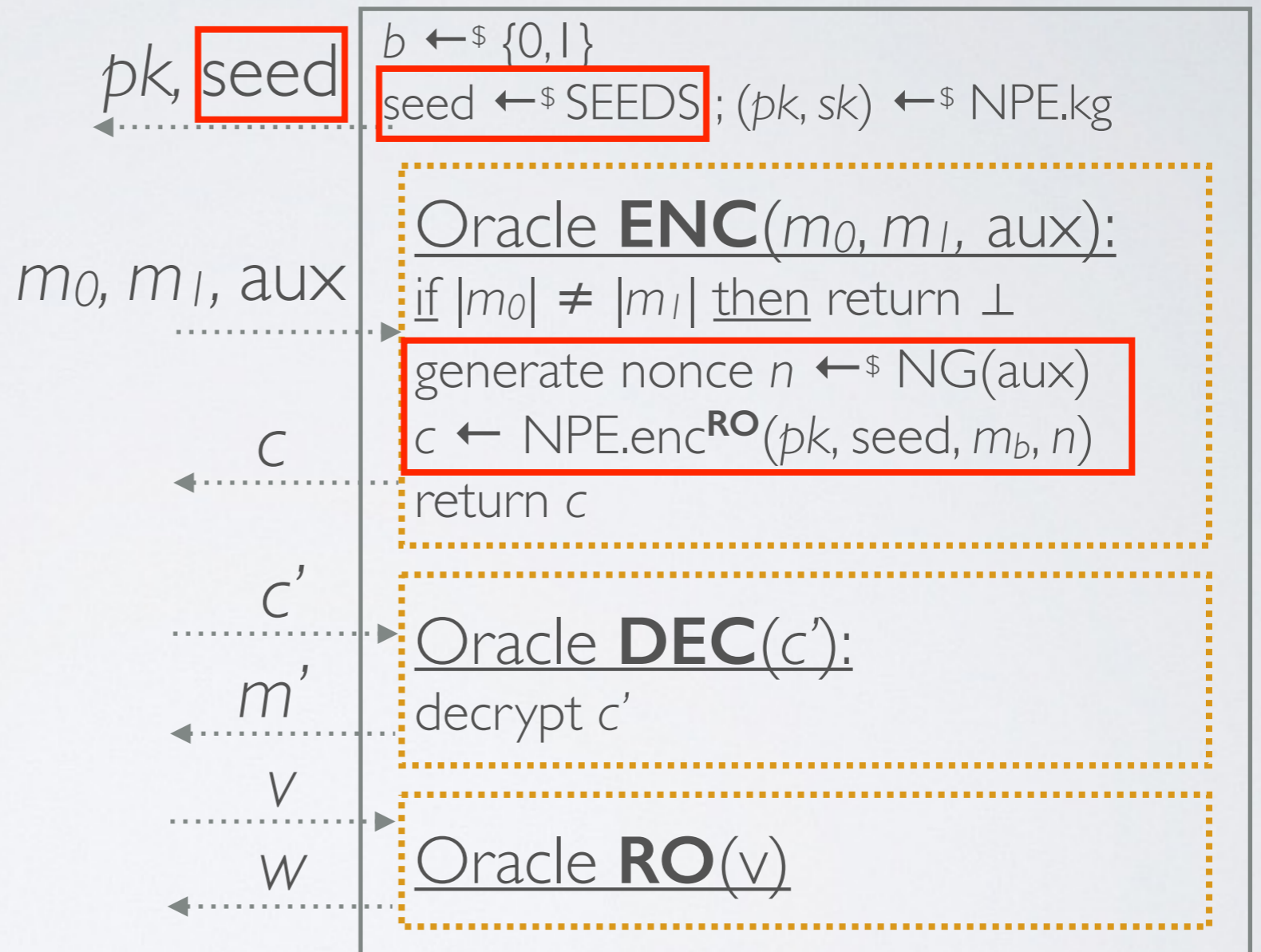
$\mathcal{A}$



$$\text{Adv}^{\text{nbp2}}(\text{NPE}, \mathcal{A}) = 2 \Pr [ b' \leftarrow_{\$} \mathcal{A}^{\text{ENC, DEC, RO}}; b = b' ] - 1$$

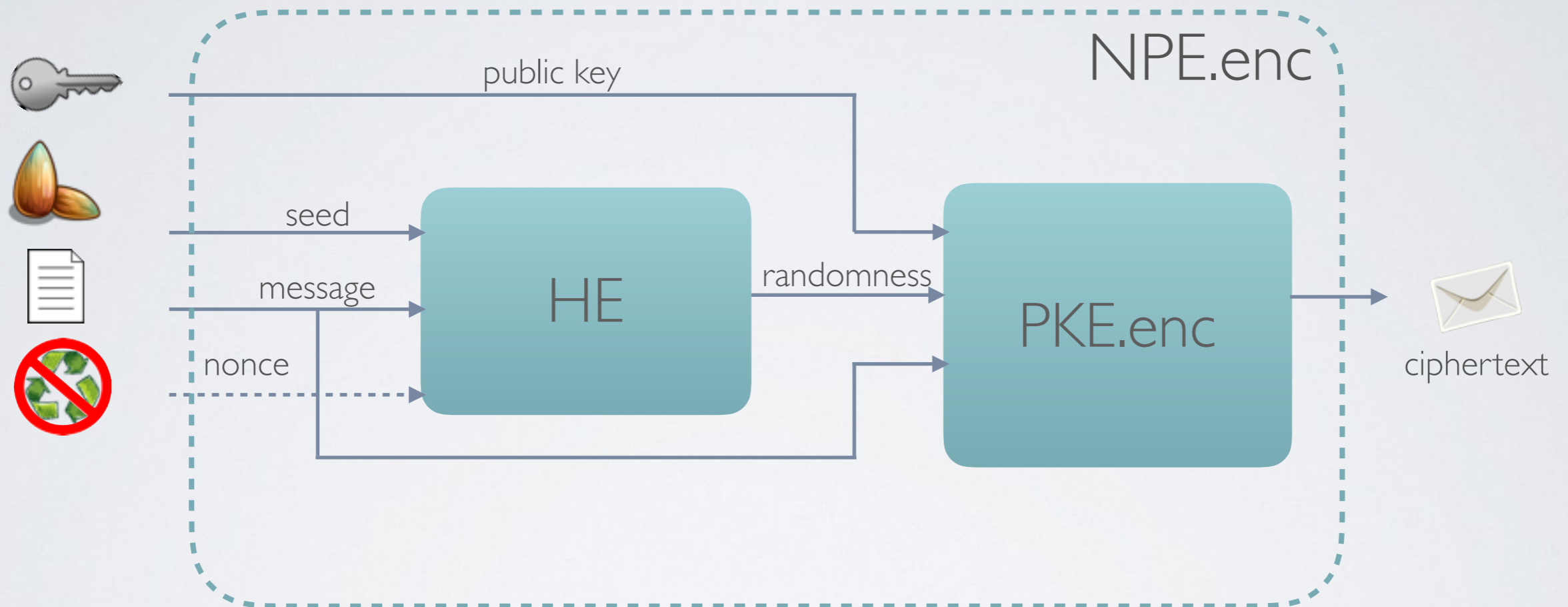
# NONCE-BASED PRIVACY, TWO

$\mathcal{A}$



$$\text{Adv}^{\text{nbp2}}(\text{NPE}, \mathcal{A}) = 2 \Pr [ b' \leftarrow_{\$} \mathcal{A}^{\text{ENC, DEC, RO}}; b = b' ] - 1$$

# BUILDING NONCE-BASED PUBLIC-KEY ENCRYPTION



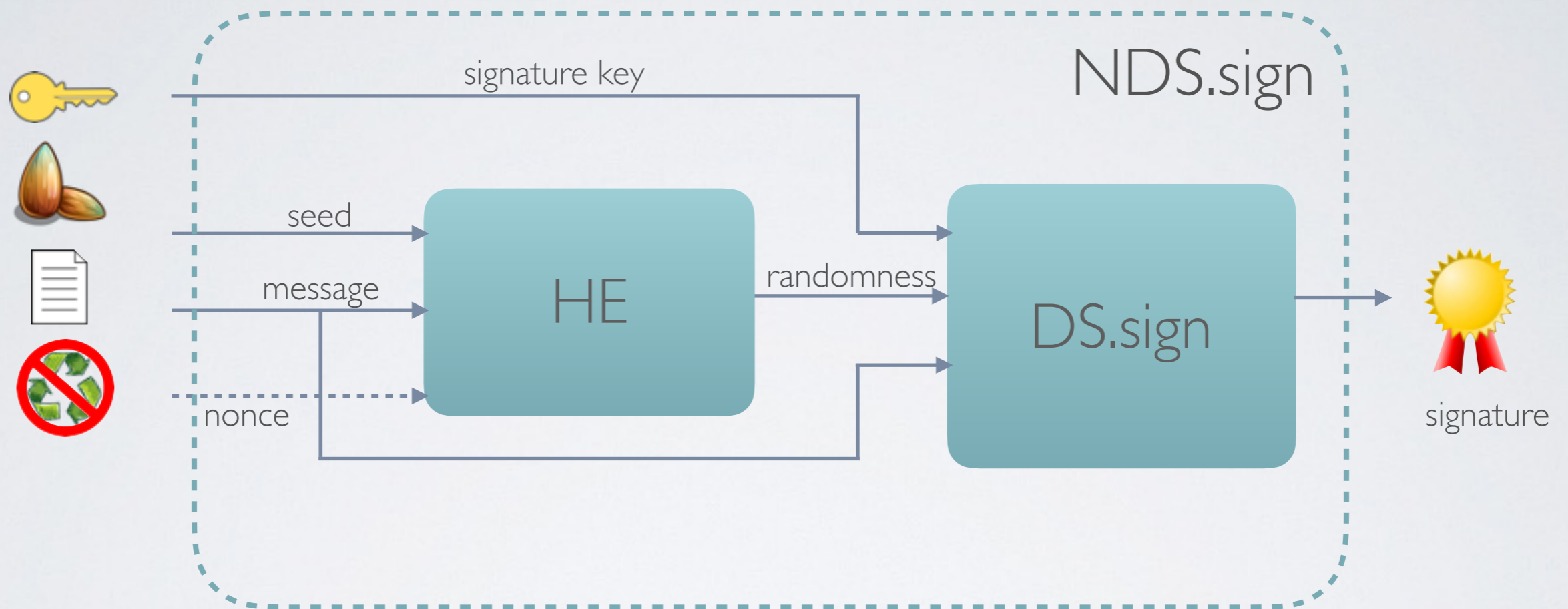
$$\text{Adv}^{\text{nbp1}}(\text{NPE}, \mathcal{A}) \leq 2 \cdot \text{Adv}^{\text{prf}}(\text{HE}, \mathcal{B}) + \text{Adv}^{\text{ind}}(\text{PKE}, \mathcal{C})$$

$$\text{Adv}^{\text{nbp2}}(\text{NPE}, \mathcal{A}) \leq 2 \cdot \text{Adv}^{\text{ror}}(\text{HE}, \mathcal{B}) + \text{Adv}^{\text{ind}}(\text{PKE}, \mathcal{C})$$

# RELATED APPROACHES

	assumption
standard pke	encryptor has access to fresh uniform randomness
deterministic pke	messages contain a certain entropy
hedged pke	message and nonce <i>together</i> have a certain entropy
nonce-based pke	seed secret, nonce unique <b>or</b> seed random, nonce entropic

# NONCE-BASED SIGNATURES



$$\text{Adv}^{\text{nbuf1}}(\text{NDS}, \mathcal{A}) \leq 2 \cdot \text{Adv}^{\text{prf}}(\text{HE}, \mathcal{B}) + \text{Adv}^{\text{uf}}(\text{DS}, \mathcal{C})$$

$$\text{Adv}^{\text{nbuf2}}(\text{NDS}, \mathcal{A}) \leq 2 \cdot \text{Adv}^{\text{ror}}(\text{HE}, \mathcal{B}) + \text{Adv}^{\text{uf}}(\text{DS}, \mathcal{C})$$

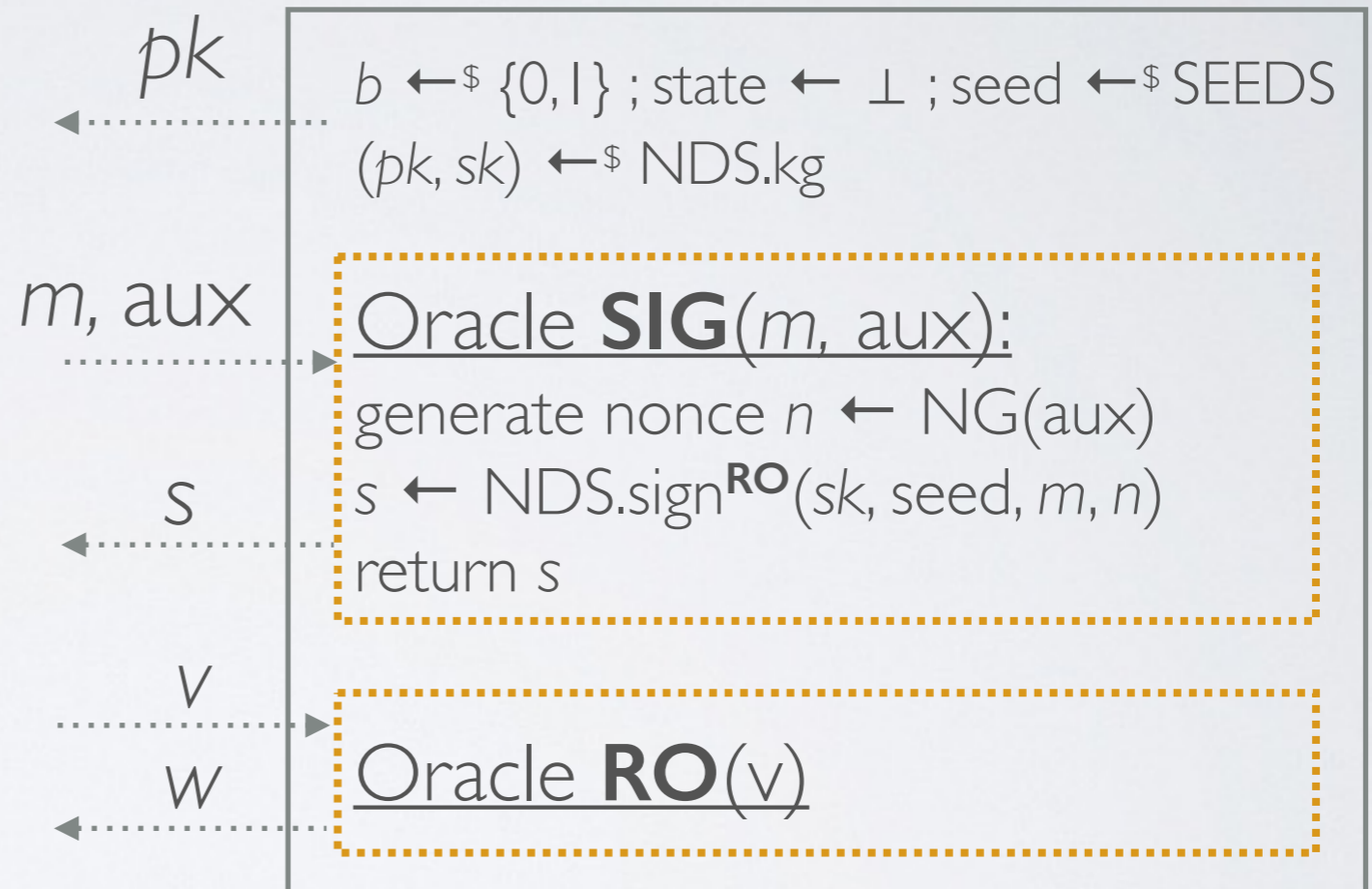


THANKS!

QUESTIONS?

# NONCE-BASED UNFORGEABILITY, ONE

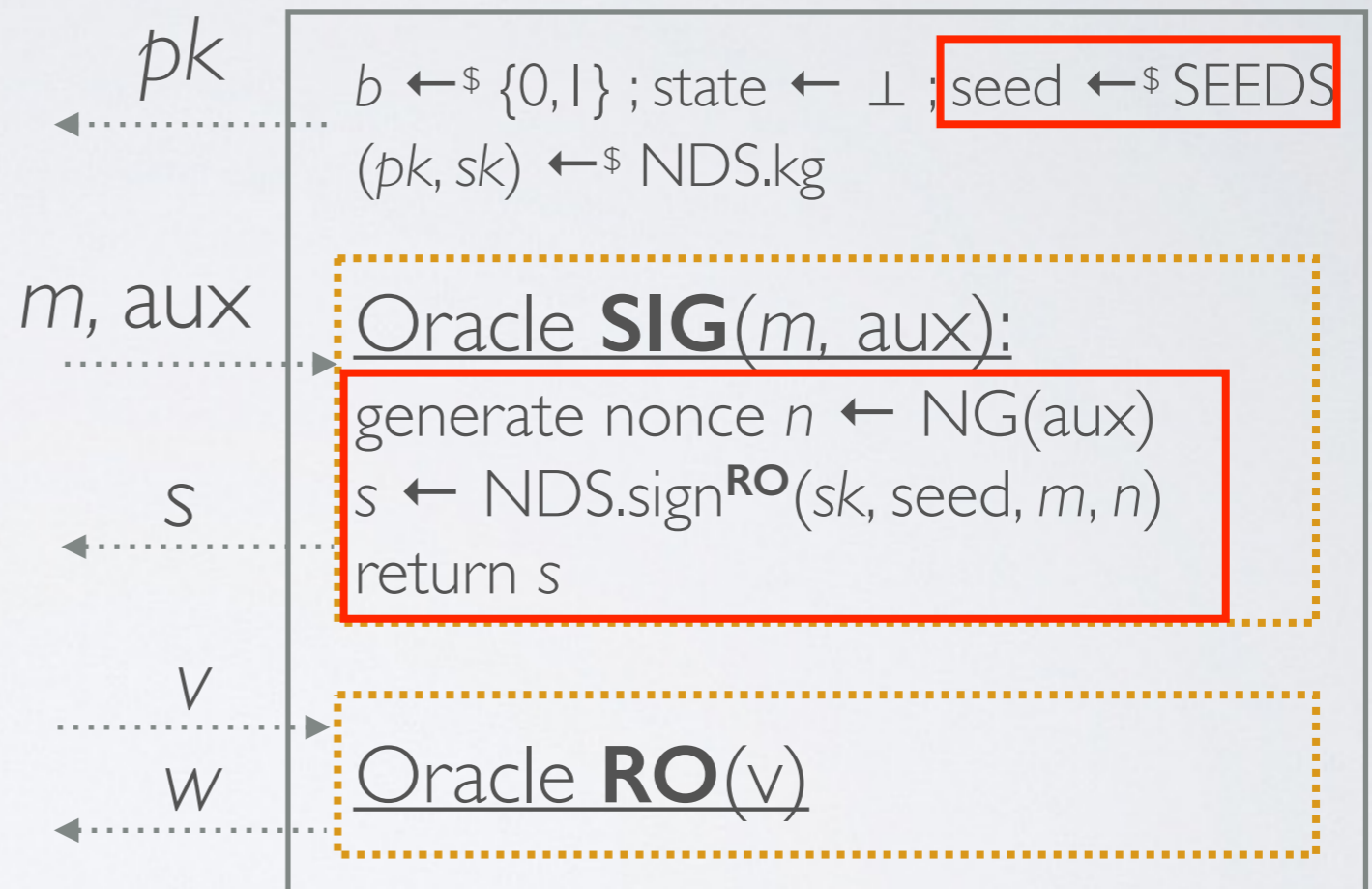
$\mathcal{A}$



$$\text{Adv}^{\text{nbuf1}}(\text{NDS}, \mathcal{A}) = \Pr [ s' \leftarrow_{\$} \mathcal{A}^{\text{SIG,RO}}; s' \text{ valid and fresh } ]$$

# NONCE-BASED UNFORGEABILITY, ONE

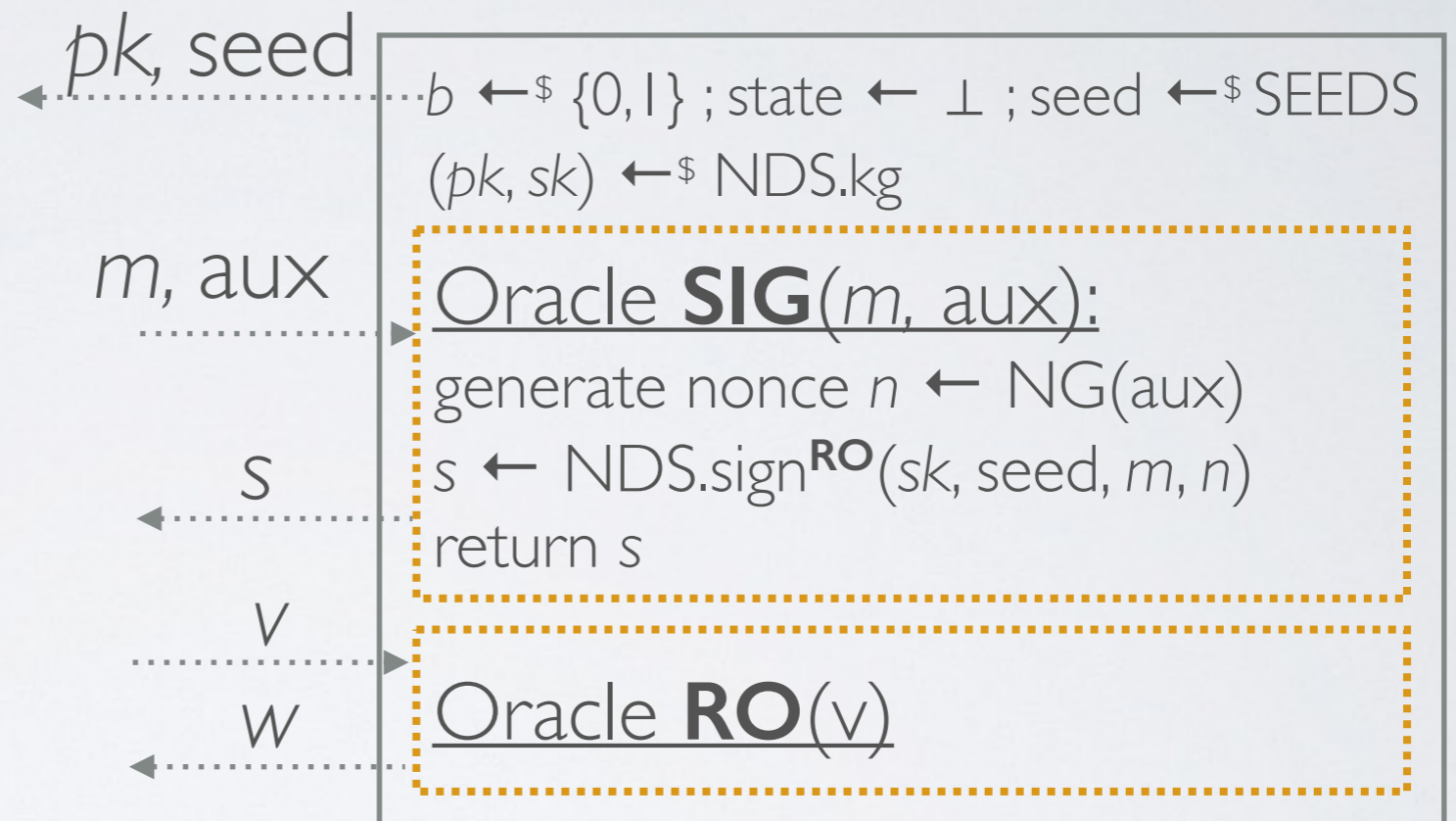
$\mathcal{A}$



$$\text{Adv}^{\text{nbuf1}}(\text{NDS}, \mathcal{A}) = \Pr [ s' \leftarrow_{\$} \mathcal{A}^{\text{SIG,RO}}; s' \text{ valid and fresh } ]$$

# NONCE-BASED UNFORGEABILITY, TWO

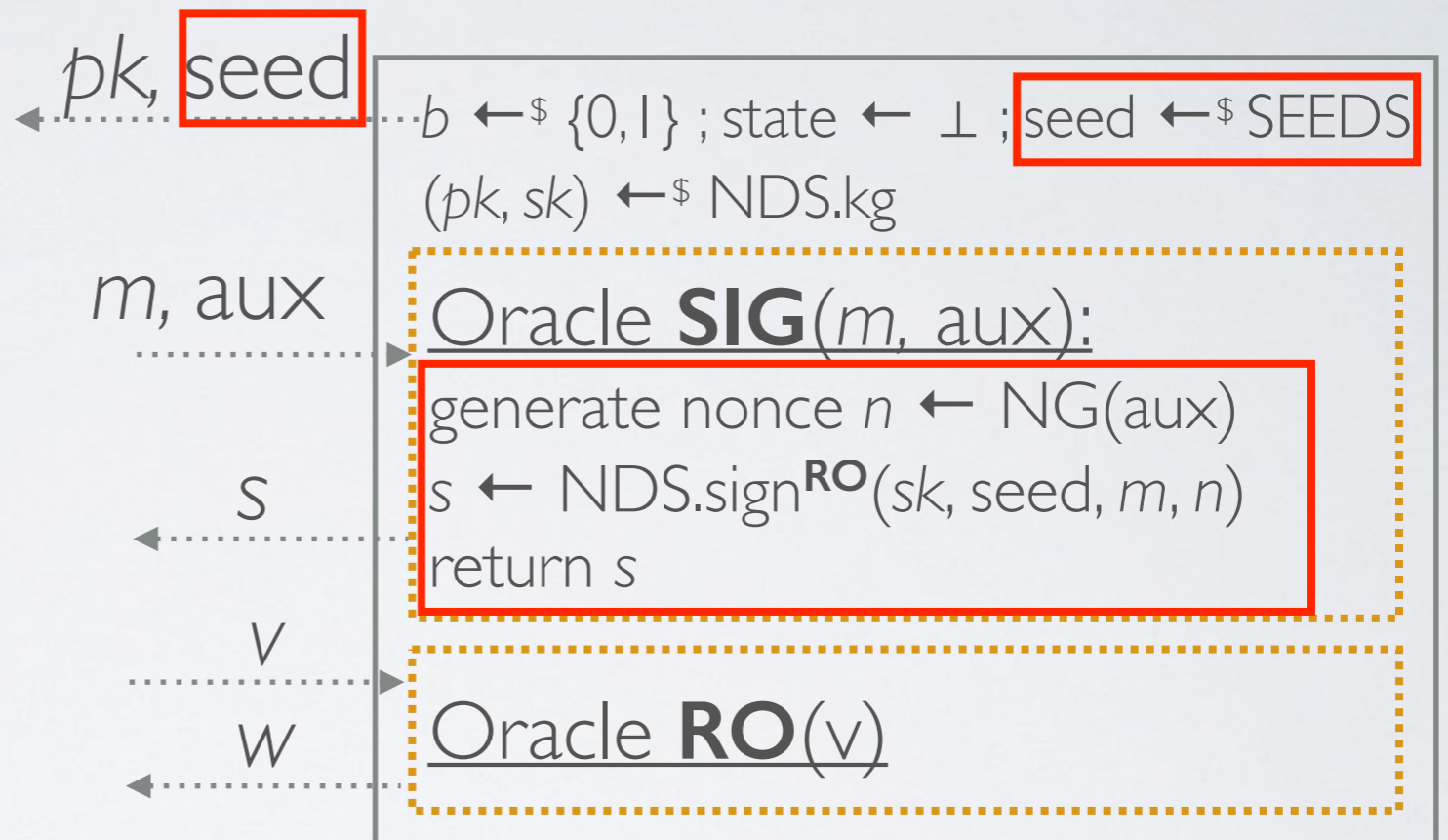
$\mathcal{A}$



$$Adv^{nbuf2}(NDS, \mathcal{A}) = \Pr [ s' \leftarrow_{\$} \mathcal{A}^{SIG,RO}; s' \text{ valid and fresh } ]$$

# NONCE-BASED UNFORGEABILITY, TWO

$\mathcal{A}$



$$\text{Adv}^{\text{nbuf2}}(\text{NDS}, \mathcal{A}) = \Pr [ s' \leftarrow_{\$} \mathcal{A}^{\text{SIG,RO}}; s' \text{ valid and fresh } ]$$